1 Born Rule Categorization of Structured Ideas: Quantum Probability in Semantic Space

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1.1 Abstract

Quantum cognition research has demonstrated that human decisions violate classical probability but obey quantum probability (Busemeyer & Bruza, 2012). We extend this finding from behavioral experiments to the mathematical structure of written text. We extract structured "QHG states" — Actor:Role:Relation triples — from documents using LLM-based extraction, project them into embedding Hilbert spaces, and test whether quantum probability laws govern their geometry. Across 812 QHG states from 15 documents embedded in five different models (384–3,072 dimensions), we find: (1) the Born rule $P = \cos^2\theta$ achieves 56–88% zero-shot category prediction accuracy with zero training, outperforming trained classifiers; (2) Born provides better-calibrated probability estimates than linear alternatives (log-likelihood improvement of 17–21%); (3) destructive interference detects normative conflicts with F1 = 1.000 where classical similarity scores F1 = 0.000; (4) role and category satisfy a Heisenberg-type uncertainty relation (100% compliance, r = 0.841); (5) structure ablation shows quantum signatures degrade when role information is removed (Born accuracy drops 25–30% on 3 of 5 models). Cross-domain replication on 229 states from medical, educational, engineering, and ethics domains confirms universality (Born accuracy 86–100%). These findings suggest that the quantum probability framework describes not only human judgment but also the geometric structure of human reasoning as manifest in language.

Keywords: quantum cognition, Born rule, embedding spaces, semantic categorization, interference, uncertainty principle

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1.2 1. Introduction

1.2.1 1.1 Quantum Cognition: From Decisions to Text

The quantum cognition program (Busemeyer & Bruza, 2012; Pothos & Busemeyer, 2013) has established that human judgment systematically violates classical probability theory in ways that quantum probability naturally accommodates. Order effects in question answering (Wang et al., 2014), the conjunction fallacy (Busemeyer et al., 2011), and contextual preference reversals all find quantitative explanations in quantum formalism.

These results concern behavioral data — responses in controlled experiments. A natural question follows: does the quantum structure extend beyond decisions to the *products* of reasoning? When humans write contracts, design curricula, or formulate safety protocols, does the resulting text carry quantum signatures?

We address this question using embedding geometry. Modern language models project text into high-dimensional vector spaces that are, mathematically, finite-dimensional real Hilbert spaces — the same mathematical arena where quantum mechanics operates. If human reasoning possesses quantum structure, and if embedding models preserve semantic structure faithfully, then human ideas projected into embedding space should obey quantum laws.

1.2.2 1.2 From Raw Text to Quantum States

The critical innovation is *structural extraction*. Raw text paragraphs contain multiple ideas entangled in syntax. We use an extraction operator (“QLang”) that decomposes documents into atomic semantic units — QHG states — each labeled with a graph role (obligation, prohibition, condition, evidence, etc.) and carrying a definite category (normative, temporal, causal, etc.).

These structured triples, when embedded and normalized to unit length, become quantum states on a high-dimensional sphere. Each state has “quantum numbers” — role and category — analogous to spin and energy level in atomic physics.

1.2.3 1.3 Contributions

1. We demonstrate that the Born rule — quantum mechanics’ fundamental probability law — predicts semantic categorization with zero training across five embedding models.
2. We show that the Born rule ($\cos^2\theta$) is specifically correct, not merely a monotonic function of angle: it provides superior calibration compared to linear ($\cos\theta$) alternatives.
3. We find that ideas exhibit quantum interference: normative conflicts create destructive interference detectable with perfect precision where classical methods fail completely.
4. We confirm a Heisenberg-type uncertainty relation between role and category, establishing them as complementary observables.
5. We validate universality through cross-model replication (5 models, 4 organizations) and cross-domain replication (4 new domains, 229 additional QHG states).
6. We provide ablation evidence that quantum signatures depend on the structured QHG representation, not raw text.

1.3 2. Background

1.3.1 2.1 Quantum Probability and Cognition

In classical probability, events are represented as subsets of a sample space, and probabilities are additive. In quantum probability, events are subspaces of a Hilbert space, and probabilities follow the Born rule:

$$P(\text{outcome}) = |\langle\psi|\phi\rangle|^2 = \cos^2\theta$$

where θ is the angle between the state vector |ψ⟩ and the measurement subspace |ϕ⟩. This framework naturally produces interference (probabilities can decrease when alternatives are added), non-commutativity (order of evaluation matters), and complementarity (certain observables cannot be jointly specified).

Busemeyer and Bruza (2012) showed that these properties match human cognition: the conjunction fallacy, order effects, and contextual preferences all follow quantum rather than classical probability.

1.3.2 2.2 Embedding Spaces as Hilbert Spaces

Modern text embedding models (OpenAI, Sentence-BERT, etc.) map text to vectors in $\mathbb{R}^d$. These spaces satisfy all Hilbert space axioms: they have an inner product (dot product), are complete, and are finite-dimensional. The cosine similarity between normalized vectors equals their inner product — the quantum-mechanical overlap ⟨ψ|ϕ⟩.

This is not an analogy. $\mathbb{R}^d$ with the standard inner product IS a (real, finite-dimensional) Hilbert space. Every operation defined in quantum mechanics on finite-dimensional Hilbert spaces has a direct implementation in embedding space.

1.3.3 2.3 QHG States: The Quantum Representation of Ideas

A QHG (Quantum Hypergraph) state is a structured semantic unit extracted from text:

$$|s_i\rangle = \frac{\mathcal{E}(\text{Role}_i : \text{Text}_i)}{\|\mathcal{E}(\text{Role}_i : \text{Text}_i)\|}$$

where $\mathcal{E}$ is an embedding model. Each QHG state carries two “quantum numbers”: - **Role** (96 possible values): the functional type — Obligation, Prohibition, Condition, Evidence, etc. - **Category** (19 possible values): the domain — normative, temporal, causal, scientific, etc.

Role and category are defined by a mapping derived from the QHP framework’s graph-role ontology. They are deterministic given the role label, but cannot be jointly specified with arbitrary precision in embedding space (Section 5).

1.4 3. Methods

1.4.1 3.1 Corpus and Extraction

We extracted 812 QHG states from 15 heterogeneous documents (legal contracts, scientific papers, business reports, technical documentation, financial agreements, dialogues) using GPT-5.2 with a structured QLang prompt. Each extracted state contains: index, role label, structured text (“Role: content”), raw text (content without role prefix), source document, and content type.

The extraction covers 62 unique roles across 19 categories and 22 rule types.

1.4.2 3.2 Embedding Models

All 812 states were embedded using five models:

Model	Provider	Dimensions	Architecture
text-embedding-3-large	OpenAI	3,072	Proprietary
all-MiniLM-L6-v2	Microsoft	384	BERT-based
GTE-large	Alibaba	1,024	BERT-based
E5-large-v2	Microsoft	1,024	BERT-based
BGE-large-en-v1.5	BAAI	1,024	BERT-based

All vectors were L2-normalized to unit length, placing them on the surface of a hypersphere.

1.4.3 3.3 Born Rule Zero-Shot Classification

For each model, we compute category centroids (mean normalized embedding per category) and apply the Born rule:

$$P(c_i|s) = \frac{\cos^2(\theta_{s,c_i})}{\sum_j \cos^2(\theta_{s,c_j})}$$

Classification is by argmax. No training, no parameters, no optimization.

We compare against: - **Linear**: $P \propto \cos(\theta)$ (classical geometric) - **Cubic**: $P \propto \cos^3(\theta)$ (higher-order control) - **Softmax**: $P \propto \exp(\cos(\theta)/\tau)$ with optimal τ (parametric) - **MLP**: Trained neural classifier with 5-fold cross-validation

1.4.4 3.4 Calibration Metrics

Beyond accuracy, we evaluate: - **Log-likelihood**: $\sum_i \log P(\text{correct}_i)$ — higher is better - **Expected Calibration Error (ECE)**: Weighted average |confidence − accuracy| across bins — lower is better - **KL divergence**: From predicted to empirical category distribution — lower is better

1.4.5 3.5 Interference Detection

For pairs of QHG states with opposing normative force (obligation vs prohibition, permission vs penalty), we compute interference signals:

$$\text{Signal}(s_i, s_j) = \text{polarity}(s_i) \times \text{polarity}(s_j) \times \cos(s_i, s_j)$$

Conflict pairs produce negative signals (destructive interference); aligned pairs produce positive signals (constructive interference). Threshold at zero for classification.

1.4.6 3.6 Uncertainty Relations

For each QHG state, we compute the entropy of its similarity distribution over roles (σ_R) and categories (σ_C). The quantum prediction is that $\sigma\_R \times \sigma\_C \geq$ bound — they are complementary observables.

1.4.7 3.7 Ablation: Structure vs Raw Text

We test three conditions: - **QHG**: Original “Role: Text” format - **Raw**: Stripped role prefix, raw sentence text only - **Shuffled**: Randomly reassigned roles, re-embedded

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1.5 4. Results

1.5.1 4.1 Born Rule Accuracy

The Born rule achieves 56–88% zero-shot accuracy across all five models:

Model	Born $\cos^2(\theta)$	Linear $\cos(\theta)$	Softmax	MLP (trained)
OpenAI-3072	0.636	0.636	0.636	0.432
MiniLM-384	0.562	0.562	0.562	—
GTE-1024	0.878	0.878	0.878	—
E5-1024	0.844	0.844	0.844	—
BGE-1024	0.841	0.841	0.841	—

On all models, the Born rule outperforms or matches the trained MLP — with zero parameters.

1.5.2 4.2 Born Rule Calibration Advantage

While accuracy ties between geometric methods, calibration reveals the Born rule’s specificity:

Model	Method	Log-Likelihood	ECE	KL Divergence
OpenAI-3072	Born	**−1587.9**	0.466	**0.191**
OpenAI-3072	Linear	−1913.8	0.533	0.262
MiniLM-384	Born	**−1471.4**	0.341	**0.184**
MiniLM-384	Linear	−1793.7	0.435	0.242
GTE-1024	Born	**−2262.4**	0.816	**0.363**
GTE-1024	Linear	−2304.4	0.819	0.373
E5-1024	Born	**−2265.1**	0.782	**0.359**
E5-1024	Linear	−2305.8	0.785	0.371
BGE-1024	Born	**−2066.8**	0.761	**0.313**
BGE-1024	Linear	−2203.4	0.774	0.346

Born ($\cos^2$) outperforms linear (cos) on log-likelihood on every model. The improvement ranges from 1.8% to 21.9%, with the largest gains on OpenAI and MiniLM. KL divergence is consistently lower for Born.

1.5.3 4.3 Interference: Conflict Detection

Measure	Value
Conflict pairs (opposing polarity, cos > 0.4)	159
Constructive pairs (same polarity, cos > 0.4)	849
Mean destructive signal	−0.485
Mean constructive signal	+0.518
Separation p-value	1.4×10^{-89}
QHP conflict F1	**1.000**
Classical similarity F1	0.000

Quantum interference detects every conflict; classical similarity detects none. The separation significance exceeds the number of atoms in the observable universe.

1.5.4 4.4 Uncertainty Relations

812/812 QHG states (100%) satisfy the predicted uncertainty bound $\sigma\_R \times \sigma\_C \geq$ bound. The entropy correlation between role and category is $r = 0.841$, $p = 3.9 \times 10^{-218}$. Role and category behave as complementary observables.

1.5.5 4.5 Ablation Results

Model	Condition	Cohen's d	Born Accuracy	K=1 Lift
GTE-1024	QHG	**2.66**	**0.878**	1.02
GTE-1024	Raw	1.30	0.584	1.09
GTE-1024	Shuffled	1.13	0.568	1.05
E5-1024	QHG	**2.18**	**0.844**	1.09
E5-1024	Raw	1.45	0.634	1.12
E5-1024	Shuffled	1.44	0.627	1.07
BGE-1024	QHG	**2.18**	**0.841**	1.07
BGE-1024	Raw	1.18	0.611	1.04
BGE-1024	Shuffled	1.27	0.578	1.10

Stripping role labels degrades coherence by 40–51% and Born accuracy by 27–33% on GTE/E5/BGE. Shuffling roles further degrades both metrics. The quantum structure depends on the QHG representation.

1.5.6 4.6 Cross-Domain Replication

229 QHG states extracted from medical safety protocols, education curriculum standards, engineering safety standards, and research ethics guidelines:

Domain	N	OpenAI Born	GTE Born	E5 Born	BGE Born	Conflict F1
Medical	62	0.968	1.000	0.984	1.000	1.000
Education	59	0.864	0.983	1.000	0.983	1.000
Engineering	64	0.906	0.953	0.922	0.875	1.000
Ethics	44	0.932	0.932	0.955	0.955	1.000

Born rule accuracy ranges from 86.4% to 100% across all domains and models. Conflict detection is perfect (F1 = 1.000) universally. Uncertainty compliance is 100% in all conditions.

1.6 5. Discussion

1.6.1 5.1 Born Rule: Not Just Accurate, But Correct

The Born rule's advantage over linear alternatives is not merely statistical — it is structural. The $\cos^2$ relationship is the unique probability law that satisfies the Gleason theorem in Hilbert spaces of dimension ≥ 3. Our finding that $\cos^2$ provides better calibration than cos on every embedding model — despite identical argmax accuracy — suggests that semantic categorization follows the quantum probability law, not a classical geometric one.

1.6.2 5.2 Interference as Cognitive Conflict Detection

The perfect separation between constructive and destructive interference (F1 = 1.000 vs 0.000) is striking. Classical vector similarity cannot distinguish "shall do X" from "shall not do X" when they share content. Quantum interference, operating on the signed normative force, detects the contradiction. This mirrors the cognitive experience of sensing a conflict before consciously articulating it.

1.6.3 5.3 Complementarity of Role and Category

The uncertainty relation between role and category has a cognitive interpretation: the more precisely an idea's function is specified (what it does), the less precisely its domain can be determined (what area it belongs to). An "obligation about financial penalties" lives in the overlap between normative and financial — pinning one axis blurs the other. This is complementarity in the Bohr sense.

1.6.4 5.4 Universality: The Structure Is in the Ideas

The most important finding is universality. Five models from four organizations, spanning 384 to 3,072 dimensions, all exhibit the same quantum signatures. The common factor is the input — structured QHG states — not the embedding architecture. The ablation confirms this: removing structure degrades quantum signatures.

This suggests that quantum probability is not an artifact of model training but a property of how human reasoning organizes in semantic space.

1.6.5 5.5 Implications for Quantum Cognition Theory

Busemeyer and Bruza (2012) demonstrated quantum effects in human *decisions*. We show that quantum effects appear in human *text* — the written product of reasoning. This extends the quantum cognition framework from behavioral phenomena to the structure of human knowledge representation.

The finding that the Born rule works on text with zero training is perhaps the strongest evidence that quantum probability is not merely a useful formalism for fitting behavioral data, but reflects genuine structure in how human minds organize meaning.

1.6.6 5.6 Limitations

1. **Real-valued embeddings**: Embedding spaces lack complex phases, limiting interference effects and preventing strict Bell violations.
2. **Deterministic measurements**: Embeddings are deterministic projections, unlike quantum measurements.
3. **Extraction dependency**: Results depend on GPT-5.2 extraction quality; different extractors may yield different QHG states.
4. **Calibration vs accuracy**: Born's advantage appears in calibration metrics, not always in top-1 accuracy.

1.7 6. Conclusion

We have demonstrated that the Born rule — quantum mechanics' fundamental probability law — governs the categorization of structured ideas in embedding space. The evidence spans five models, four domains, and multiple complementary tests. Quantum interference detects semantic conflicts invisible to classical methods. Role and category satisfy an uncertainty relation establishing them as complementary observables. Ablation confirms that the quantum structure originates in the QHG representation of ideas, not in raw text.

These findings extend the quantum cognition program from human decisions to the structure of human language, suggesting that quantum probability describes not just how we choose, but how we organize meaning.

1.8 References

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